COMMUNICATIONS

A POSSIBLE FALLACY IN THE USE OF THE CROSS-CYLINDER*

BY

F. A. WILLIAMSON-NOBLE

LONDON

Nearly 50 years ago Edward Jackson described the cross-cylinder test for determining, subjectively, the amount of astigmatism present in a patient, and some 14 years later, he described how it could also be used to determine the axis of the astigmatism.

It was a long time before the value of Jackson's teaching was generally appreciated and as recently as 1931, William Crisp,¹ in an address to this Congress lamented the fact that even in the United States, the use of the cross-cylinder was really familiar only to some 5 per cent. of ophthalmologists, estimating that the percentage was lower in this country and lower still on the continent.

Incidentally, as so often happens, it was probably a denizen of the (then) United Kingdom who first discovered this useful piece of glass, and Crisp records how Jackson informed him that in 1849, Gabriel Stokes, an Irish mathematician, later Professor of Mathematics at Cambridge and President of the Royal Society, described the general principles of crossed cylinders. At that time the device was known as the Stokes lens and although it is probably too late now, it would seem fitting that the name should be revived. Ophthalmology contains many examples of the same sort of thing, for example, Guthrie's section, misnamed Saemisch's; the Von

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Hippelsche Krankheit, which should be Treacher Collins' disease, and least known perhaps, the Foster Kennedy syndrome which was first described by Leslie Paton to Gowers and mentioned by him in a lecture at Queen Square.

I am not at the moment, however, concerned with correcting the nomenclature of diseases and syndromes, though perhaps at a future meeting of this Congress, someone intimately acquainted with the historical side of our speciality could give us an entertaining paper on this subject.

To return then to Dr. Crisp—after lamenting the paucity of users of the cross-cylinder, he proceeded to give a very full and useful account of its applications. These are so well known now that I need not detain you with them, but there is one statement made by Dr. Crisp which may lead to inaccuracies in the test, and as this is the subject of my paper, I shall quote his statement in full. It goes as follows: "Like all other astigmatic tests, this one is usually more likely to be successful if the accommodation is relaxed, and, therefore, if whatever spherical lens is in the trial frame is so strong a plus or so weak a minus as barely to allow the patient to obtain his full visual acuity."

Now in my experience, this is not so, and I have found, in common probably with most of you, that if the patient is fogged, he tends to choose too big a cylinder against the rule, whereas in the opposite condition, i.e. when his hypermetropia is undercorrected, or his myopia overcorrected, he tends to choose too big a cylinder with the rule. The dilemma does not of course occur to the same extent when the axis of the correcting cylinder is oblique, and may be entirely absent when it is at 45° or 135°.

There are two factors which are responsible for its occurrence. The first of these is that blurred letters are more easily recognisable when their vertical parts are relatively in focus than when their horizontal are.

The same is true of sailors spotting ships at sea—quite a number of particularly good spotters are found to have astigmatism with the rule, and to be less good when this is corrected, because then there is no longer so much contrast between a vertical mast, funnel, or periscope, and the slightly blurred horizon.

Nature is said to make use of this in the seal's eye. The world above the water is largely a world of vertical lines, whereas under the water it is largely one of horizontal lines, and the seal's eye is highly astigmatic in the vertical meridian. The amount has been so chosen that above the water there is astigmatism corresponding with what would be produced by adding a plus cylinder axis horizontal, thus rendering vertical lines sharper than horizontal, whereas below the water the optical effect of the latter, makes horizontal lines clearer than vertical because under these conditions the eye becomes
virtually hypermetropic and so sees better in the more highly corrected meridian.

This may be a fable and if Miss Mann is here, perhaps she will tell us if it has any substratum of truth. If it has, it provides rather a striking example of adaptation to environment and of the cunning way in which Nature takes advantage of apparent defects in bodily organs.

The majority of our patients, however, are not seals, nor are they mariners looking out for masts at sea. They come to us in the hope of being rendered emmetropic by glasses, and if we are not careful, we may send them away with an overcorrection for astigmatism.

Mr. Sands, of Messrs. Hamblin, has kindly taken a series of photographs under my direction which show how this may occur when using the cross-cylinder.

1. The first series shows the effect of fogging.

A represents the familiar 6/9 line (so often miscalled nowadays A R P instead of A N P), as seen by an emmetrope.
B the appearance when a +0·50 sphere is added, i.e. when it is slightly fogged.

C shows the effect of placing a ±0·50 crossed cylinder in front with the + axis horizontal, and

D the effect of rotating it so that the − axis is horizontal.

I think that most people would choose C as being more legible than D in spite of the fact that the "eye" is question (i.e. the photographic lens) has no astigmatic error.

2. In No. 2 we did the opposite thing, i.e. we added a −0·50 sphere with the result shown in B. Here D is more legible than C,

i.e., the patient whose hypermetropia is undercorrected or whose myopia is overcorrected, would tend to prefer a +cylinder axis vertical even in the absence of an astigmatic error.

3. This is a repetition of 1, but with only a +0·25 sphere added and a cross-cylinder of ±0·25. The same thing is evident, though of course not so markedly, i.e. the correction against the rule is preferred.
No. 3.

A. ANPWFZ
   TYPE IN FOCUS.

B. ANPWFZ
   WITH SPH. +0.25 ADDED.

C. ANPWFZ
   WITH SPH. +0.25 ADDED AND CROSSED CYL. (±0.25) + AXIS

D. ANPWFZ
   WITH SPH. +0.25 ADDED AND CROSSED CYL. (±0.25D) + AXIS

No. 4.

A. ANPWFZ
   TYPE IN FOCUS.

B. ANPWFZ
   WITH SPH. -0.25 D. ADDED.

C. ANPWFZ
   WITH SPH. -0.25 D. ADDED AND CROSSED CYL. (±0.25D) + AXIS

D. ANPWFZ
   WITH SPH. -0.25D. ADDED AND CROSSED CYL.(±0.25D) + AXIS
4. This is a repetition of 2; but with a −0.25 sphere added, and here as in 2 I think it would be admitted that a +cylinder axis vertical gives the better vision.

After seeing these photographs, you may begin to wonder if the cross-cylinder really does work, so here is a representation of what is seen by an emmetropic eye.

5. A is the type in focus.
   B is the appearance with the ±0.5 cross-cylinder with the +axis horizontal.
   C is the appearance with the ±0.5 cross-cylinder with +axis vertical, and I think you will agree that there is nothing much to choose between B and C.

This is not the whole story, however, because so far we have been considering only the fallacies which can occur in an emmetropic eye, and have shown that we can make it prefer a +cylinder axis horizontal or vertical according to whether a convex or a concave spherical lens is added.

When the eye is already astigmatic, the bad effects of fogging again manifest themselves, and as has been pointed out by Pascal,² can be deduced by working out the sizes of the diffusion circles on
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the retina. They also show photographically and here is a series which demonstrates this. At the end of each row, I have drawn a figure to show the corresponding sizes and shapes of the diffusion circles on the retina which would be produced by the combination of lenses employed. This has been done by using an arbitrary scale to represent the fractions of a dioptre by which a point source of light would be out of focus. Since we are dealing with only fractions of a dioptre, it has been assumed that the size of the diffusion circle produced in an eye with say half a dioptre of myopia is the same as that produced in one with half a dioptre of hypermetropia, cycloplegia being of course assumed to be present.

FIG. 6. Here are the photographs. In Figs. 6; 7, 8 and 9 we are dealing with an eye which has −0.5D. of astigmatism with the rule, i.e., the camera was first focused accurately and a +0.5D. cylinder axis horizontal was placed in front of the lens. In this (Fig. 6), a +0.5D. sphere was also placed in front of the camera lens, thus producing the effect of slight "fogging," as shown in A.

In B a ±0.25 cross-cylinder has been added with the plus cylinder horizontal. In C, it has been rotated so that the + cylinder is
vertical. As shown by the diffusion diagrams, the patient has to compare letters made up of superimposed elongated ovals measuring 1.25 by 0.25D. with those made up of circles measuring 0.75 by 0.75D. It would not be very easy to choose between these and some patients might even prefer B, giving a totally incorrect idea of the refraction.

**Fig. 7.** In this figure the fogging has been reduced to 0.25D., and again the choice between B and C would be difficult and some patients might prefer B, particularly for the letters N and U.

**Fig. 8.** In this, the eye is only half fogged, *i.e.*, one focal line is on the retina, the other in front, and here the difference between B and C is more marked, C being obviously the better and so indicating the right answer.

**Fig. 9.** Here the condition is changed to one of mixed astigmatism, one focal line being 0.25D. in front of the retina, the other 0.25D. behind, and there is no question as to whether B or C is the better. The reason for this is shown by the size of the diffusion circles, which in B measure 0.50D. by 0.50D., whereas in C they are
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No. 8.

A. ANPUFZ 1.5 x 0

Eye half fogged i.e., one focal line on retina, the other 0.5D in front (+0.5D cyl. →)

B. ANPUFZ 0.75 x 2.5

With crossed cyl. (±0.25D) + axis added to A.

C. ANPUFZ 0.25 x 2.5

With crossed cyl. (±0.25D) + axis added to A.

Change more definite

No. 9.

A. ANPUFZ 0.25 x 2.5

Condition changed to mixed astig.

i.e., one focal line in front and the other behind the retina. (Sph. -0.25 & cyl. +0.50D axis →)

B. ANPUFZ 0.50 x 50

With crossed cyl. (±0.25) + axis added to A.

C. ANPUFZ 0.0 x 0

With crossed cyl. (±0.25D) + axis added.

Difference striking
point foci. At first sight it may seem rather odd that the diffusion spot in B should be circular when the eye has actually 1D. of astigmatism, but a little arithmetic soon shows that this is so. The lens before the camera is a sphere $-0.25$ with a cylinder $+0.50$ axis horizontal; writing this in the way usually adopted in retinoscopy we have

\[
\begin{align*}
\text{lens in front of camera. Sphere } &-0.25 \\
\text{with cylinder } &+0.50 \\
\text{axis horizontal.}
\end{align*}
\]

The effect of adding a $\pm 0.25$ crossed cylinder with the $+$ axis vertical is to produce

\[
\begin{align*}
-0.25 &+0.25 \\
+0.25 &-0.25
\end{align*}
\]

(3) shows the effect of adding $\pm 0.25$ cylinder with $+$ axis vertical.

\[
\begin{align*}
\text{i.e., zero, whereas with the plus axis horizontal we get}
\end{align*}
\]

\[
\begin{align*}
-0.25 & \quad -0.25 \\
+0.25 & \quad +0.25
\end{align*}
\]

(4) shows the effect of adding $\pm 0.25$ cylinder with $+$ axis horizontal.

\[
\begin{align*}
\text{i.e., an image in which one focal line is half a dioptre in front and}
\text{the other half a dioptre behind the retina, producing therefore a}
\text{diffusion circle on the retina half a dioptre in diameter.}
\end{align*}
\]

**FIG. 10.** This shows the effect when not all the astigmatism is corrected by the crossed cylinder.

In A we see the image produced when there is $0.75$ D. of mixed astigmatism, while B and C show the effects of twirling the crossed cylinder, and they are sufficiently obvious not to need any comment, even though C is made up of diffusion circles $0.12$ D. in diameter instead of by point foci.
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No. 10.

A. ANPUFZ
MIXED ASTIG. -75 D. (SPH. -0.37 D. CYL. +0.75D →)

B. ANPUFZ
WITH CROSSED CYL. (± 0.25D) + AXIS →
ADDED TO A.

C. ANPUFZ
WITH CROSSED CYL. (± 0.25D) + AXIS ↓
ADDED TO A.
DIFFERENCE STILL STRIKING EVEN THOUGH WHOLE ASTIGMATISM NOT CORRECTED.

Having shown that these defects occur and it is a very simple matter to convince oneself that they do, either subjectively or with a patient, how are we to overcome them? In the first place, their occurrence emphasises the need for an accurate retinoscopy, since the performance of this will prevent the ophthalmic surgeon being led too far “up the garden path.” Assuming this to have been done, I find that the duochrome letter test is of great assistance in keeping the astigmatism mixed. I usually check up on it now and then as one increases or diminishes the cylindrical correction during the course of the cross-cylinder tests, adding + or — spheres as may be required to make the letters on the red and green equally defined, or rather, blurred. I then return to the black and white letters for comparison of the two positions of the cross-cylinder. Another check which is often useful, is a Tweedy Cross, or some modification of it, which is set in the axis of the correcting cylinder. The eye of course has to be fogged 0.50 D., or so, and one can then so to speak bracket with the cross-cylinder if the cylindrical correction is right, if not, an indication will be obtained as to how it should be changed.

Two other methods are possible, but as I have not used them
much—having only thought of them when writing this paper—I can do no more than suggest them.

One depends on the observation that this fallacy is not so evident when the axis of the correcting cylinder is at 45° or 135°. If therefore rotating test types could be produced, it might be worth while to tilt the letters so that their axes were at 45° to the axis of the patient's astigmatism, and use them in this position for subjective testing.

The other test is very simply performed, and it seemed to work when I tried it on myself and on a few patients, though I have never seen it described. When the final correction has been obtained and the letters are equally clear on the duochrome with the glasses in the trial frame, a ±0·25 cross-cylinder is added, and it will be found that the letters on the red background are clearer when the +cylinder is vertical, and those on the green background when it is horizontal. This is what one would expect because with both lots of letters slightly and equally blurred, the green image is in front of the retina and the red behind, i.e., the eye is relatively myopic for the green and therefore sees better with a +cylinder horizontal than with it vertical. So far as the red is concerned, however, the eye is relatively hypermetropic for it and therefore prefers the +cylinder axis vertical.

This is not meant as a test for the amount of astigmatism present; it is only employed as a check when the patient has his supposedly final correction in the trial frame and even then need only be employed when the axis of the correcting cylinder is within about 20° of the vertical or horizontal meridian.

In conclusion, I hope you will not think me guilty of adding complications to a test which has simplicity as its great merit. I can only plead that in the final analysis refraction is an art and not a science. Experience with actual cases teaches us far more than any books or papers can do. We therefore welcome any extra help we can get in the difficult task of trying to reduce to figures the deformities of a lens system which has not been turned out on an optician's grinding machine, but has grown in such an apparently haphazard manner that we marvel at how nearly it approaches what should be its ideal dimensions.

Finally, I should like to record my gratitude to Joseph Pascal of New York, whose pioneer work in this field has been the origin of this paper.

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