Mathematical analysis and experiment on the corneal reflex test in spectacle wearers

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Abstract

An experimental study and mathematical analysis of the corneal reflex test was undertaken in spectacle wearers. In the experimental study, photographs were taken of the corneal reflex through spectacles and the conversion ratios determined as measured in $^{\circ}/\text{mm}$. In the mathematical analysis, the magnification effect of the lens was elucidated by three methods: geometrical analysis; real measurement of magnification factor; and ray tracing analysis. The real measurement of the conversion ratios was in good agreement with the conversion ratios determined by the three mathematical analyses. These results clearly showed that the corneal reflex test can be clinically useful even in wearers of spectacles.

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The corneal reflex test (CRT) (Hirschberg test¹) is one of the most important examinations in determining the angle of deviation of squint, especially in infants or non-verbal children. The CRT has been re-evaluated in recent papers, and new conversion ratios of 21-22 prism dioptres/mm or 12-14 $^{\circ}/\text{mm}$ ³⁻⁷ were proposed. However, most squint patients wear spectacles, and it is important to have information on the angle of deviation from the CRT in these spectacle wearers. As yet few investigations⁸⁻¹⁰ have been carried out on the CRT in such patients.

We have undertaken an experimental study and mathematical analysis of the CRT in spectacle wearers. We analysed mathematically the magnification factor using geometrical analyses, real measurement, and ray tracing

analysis. We compared the experimental measurements with the mathematical measurements. The results were in good agreement. As far as we know, this is the first approach using mathematical analysis of the CRT in spectacle wearers.

Materials and methods

EXPERIMENTAL STUDY

Four emmetropic subjects aged 35 to 45 years were examined. All of them looked at the targets at 0° to 25° nasally or laterally at each 5° without and with spectacles. The spectacles had +12D, +9D, +6D, +3D, -3D, -6D, -9D, and -12D plano convex or plano concave test lenses. The plane surface was towards the cornea.

The photographic apparatus consisted of a flash aligned 1 metre in front of the centre of both eyes of each subject. A camera with a 210 mm telescopic lens was used, and the corneal reflexes of the flash without and with spectacles were photographed using Ektachrome ASA 200 film. The slides were projected onto a screen, and the distances between the corneal reflexes and the centre of the pupil without and with spectacles were measured on the screen. As the lens has a prismatic effect of its own, the true angle of rotation of the eye was devised from the formula¹¹ (Fig 1);

$$\left(\frac{\Delta_r}{\Delta_m}\right) = 1/(1 - 0.025 D) \quad (1)$$

D = lens power

The true angles of rotation of the eye and the distances between the corneal reflex and the centre of the pupil were plotted, and the regression lines were determined in mm/ $^{\circ}$ or $^{\circ}/\text{mm}$ (Fig 2A, B, C). The coefficient of correlation (γ) ranged from 0.978 to 0.999. The measurements in $^{\circ}/\text{mm}$ were considered to be conversion ratios of the CRT through spectacles and were compared with the conversion ratios determined by the mathematical analysis.

MATHEMATICAL ANALYSIS

Mathematical analysis was carried out on the magnification effect of the spectacles. The magnification factor was studied by three methods: (a) geometrical analysis, (b) measurement of magnification factor, and (c) ray tracing analysis. These analyses were used to determine the magnification factor through spectacles. As 1 mm without spectacles becomes $1 \times m$ millimetres through spectacles (m = magnification factor), a conversion ratio of ($^{\circ}/\text{mm}$) can be transformed to a conversion ratio of ($^{\circ}/\text{mm}$) through spectacles if divided by $1 \times m$. For example, if the magnification factor was m and

Figure 1 Prismatic effect of lenses. When the deviation of the eye was Δ_m prism dioptres through the lens, the true rotation of the eye was Δ_t prism dioptres. The distance between lenses and the centre of rotation of the eye was 25 mm. From Scattergood *et al.*,¹¹ reproduced with permission.

Fig 2A

Figure 2 The relation between the corneal reflex and the rotation of the eye without and with spectacles of -12D, -9D, -6D, -3D (A), without spectacles, +3D, +6D (B), and +9D and +12D (C). The regression line and the coefficient of correlation is shown in each graph. The conversion ratios change with the dioptries of the lens.

a conversion ratio without spectacles was x (°/mm), the conversion ratio through spectacles was determined as x/m (°/mm). Details are explained in the results. The real distances between the corneal reflex and the centre of the pupil without spectacles were divided by the magnification factors using these three methods, and were compared with the real distances between the corneal reflex and the centre of the pupil through spectacles.

Geometrical analysis

We have introduced a model eye. The following assumptions were adopted: (1) the angle λ (the

angle between the principal line of vision and the central principal line) of the model eye was 0° ; (2) the centre of the pupil, the centre of the corneal curvature, and the centre of rotation of the eye were on the same line.

The virtual image of the eye was made by the lens (Fig 3A).

In the model eye, when the virtual image of the eye was considered, the corneal reflex was imaged geometrically in the same way as it was imaged in the real eye without spectacles, and the position of the corneal light reflex was determined in the virtual image of the eye. In Figure 3B, when the eye was rotated θ° , so the virtual image of the eye rotates θ° , the distance between

Fig 2B

the corneal reflex in the virtual eye (LR') and the centre of the entrance pupil of the virtual image of the real eye (p') was magnified according to the magnification factor of the lens. The magnification factor (m) is as follows^{9 12}:

$$m = 1 - dL / 1 - d(L + F) \quad (2)$$

d = the distance between the lens and the observer's eye (m)

L = the reciprocal of the distance between the lens and the entrance pupil of the real eye

F = power of the lens (diopetre)

Example: If $d = +1.0$, $L = -1/0.015$, $F = +6.0$, then the magnification factor is 1.097.

As the length from the corneal surface to the centre of rotation of the real eye is larger than the radius of the corneal curvature, when the real eye rotates, the centre of the corneal curvature

Figure 3 (A) The virtual image of the eye made by the lens. E =real eye, E' =virtual image of the real eye. (B) The rotation of the eye and the corneal reflex made in the virtual image of the eye. L =light source, F =the anterior focus of lens, θ =angle of rotation, LR' =the corneal reflex made in the virtual image of the eye, CC' =centre of corneal curvature of the virtual image of the eye, p' =distance of the corneal reflex from the centre of pupil of the virtual image of lens.

Fig 2C

always shifts laterally from the optical axis of the lens on which the light source exists (Fig 4). In Figure 4, if $y = 13 \text{ mm}$, $\theta = 25^\circ$, and the radius of the corneal curvature is 7.80 mm , then $x = 2.2 \text{ mm}$. If the power of the lens is $+6.0 \text{ D}$, the virtual image of the centre of the corneal curvature is calculated as 10.45 mm behind the corneal surface of the real eye. So, if a light source is 1000 mm in front of the corneal surface of the real eye and the centre of the corneal curvature of the virtual image of the eye shifts 2.2 mm laterally, the corneal reflex in the virtual image of the eye moves laterally at angle δ . The angle δ is calculated as $\tan^{-1} 2.2/1000 + 10.45 = 0.12^\circ$. So, the angle δ can be neglected.

Real measurement of the magnification factor

The cornea was photographed without spectacles and with $+12\text{D}$, $+9\text{D}$, $+6\text{D}$, $+3\text{D}$, -3D , -6D , -9D , and -12D spectacles. The slides of the cornea without and with spectacles were projected onto the screen, and the horizontal diameters of the cornea without and with spectacles were compared. The magnification factor (m') was determined as;

m' =the corneal diameter through spectacles/
the corneal diameter without spectacles

Figure 4 The rotation of the eye and the lateral shift of the centre of the cornea and the centre of the corneal curvature. $x = y \times \sin \theta$ and $x' = (y - r) \times \sin \theta$. x = the lateral shift of the centre of the cornea; x' = the lateral shift of the centre of the corneal curvature, y = the length from the corneal surface to the centre of rotation of the eye, r = the radius of the corneal curvature.

Ray tracing analysis

The position of the corneal reflex through spectacles was determined by computer analysis using the ray tracing method. In the ray tracing analysis, the following factors were input to computer analysis. (1) The vertex distance, (2) the radius of the corneal curvature of the real eye, (3) the estimated position of the centre of rotation, and (4) the refractive index of the anterior chamber fluid for the light of wave length 550 nm (1.334). The vertex distance was

determined by measuring directly the length from the posterior surface of the plano convex or plano concave lens to the corneal surface. The radius of the corneal curvature was measured by the automatic keratometer (Canon RK-2), and the radius of the horizontal meridian (r_{180}) was determined from the formula;

$$r_{180} = 1 / (1/ra + (1/rb - 1/ra)(\sin \alpha)^2)$$

a = first meridian of the astigmatism

b = second meridian of the astigmatism

$a = b + 90$

α = the angle between b and 180°

Results

The conversion ratios ($^\circ/\text{mm}$) determined by the real measurement of the corneal reflex with each lens in the four subjects were shown by the symbol ($\circ$) in Figure 5. The mathematical results were shown by symbols ($\blacktriangle$), ($\triangle$), and ($\blacksquare$). If subject 1 is taken as an example, the conversion ratio ($^\circ/\text{mm}$) without spectacles determined from real measurement of the corneal reflex was 11.8 ($^\circ/\text{mm}$). The magnification ratio determined by the geometrical analysis using the equation (2) was 1.06 with +3D, 1.13 with +6D, and 0.945 with -3D; so, when 11.8 ($^\circ/\text{mm}$) was divided by 1.06 with +3D we get 11.1, 1.13 with -3D we get 10.5, and 0.945 with -3D we get 12.4. The conversion ratios ($^\circ/\text{mm}$) with other

Figure 5 The result of real measurement determined by photograph was shown by the line ($\circ$ — $\circ$), the ratios determined by the geometrical analysis ($\blacktriangle$ — $\blacktriangle$), the ratios determined by the real measurement of the magnification factor ($\blacksquare$ — $\blacksquare$).

Figure 6 The Hirschberg test is as useful when wearing spectacles as it is without. ET=esotropia.

dioptries of lenses were calculated in the same way and plotted in Figure 5A ($\blacktriangle$). The magnification factor determined from the comparison of the corneal diameter without and with spectacles was 1.07 with +3D, 1.12 with +6D, and 0.947 with -3D; so, when 11.8 ($^{\circ}$ /mm) was divided by 1.07 with +3D we get 11.0, 1.12 with +6D we get 10.5, and 0.947 with -3D we get 12.4. The conversion ratios ($^{\circ}$ /mm) with other dioptries of lenses were also calculated in the same way and plotted in Figure 5A ($\triangle$). By ray tracing analysis, the conversion ratios with spectacles were determined with computer analysis, and were plotted in Figure 5A ($\blacksquare$). The same procedures were done in subjects 2, 3, and 4 and the results are shown in Figure 5B, C, and D.

The real measurement line ($\bigcirc$ — $\bigcirc$) of the conversion ratio through spectacles was in good agreement with other lines of the conversion ratios ($\blacktriangle$ — $\blacktriangle$, $\triangle$ — $\triangle$, $\blacksquare$ — $\blacksquare$) determined by mathematical analysis on all subjects.

Discussion

The corneal reflex test is frequently used in clinical practice in patients wearing spectacles. It is important to estimate the angle of deviation in the normal viewing conditions of spectacle wearing. However, only a few papers have studied the corneal reflex test with spectacles. We have attempted to deal with this problem.

Firstly, we studied experiments by taking pictures of the corneal reflexes through spectacles and measuring the real distances between the corneal reflexes and the centre of the pupil. In the real measurement, we considered that the true angle of rotation of the eye must be revised by the prismatic effects of the spectacles calculated from equation (1). For example, when the subject looked at the target of 20° laterally through a lens of +6.0D, the true angle of rotation of the eye was calculated as 22.7° (11.4% larger). The regression line was determined from the plotted data of the true rotation of the eye and the true position of the corneal reflex.

In the mathematical analysis, we introduced a model eye. The mathematical analysis was made with this model eye and the magnification factor was determined. With geometrical analysis, it was shown that the corneal reflex through spectacles was imaged in the virtual image of the real eye as if it were imaged in the real eye without spectacles. It was also shown that the side shift of the cornea during rotation of the eye did not much affect the position of the corneal reflex. The real measurement line of the conversion ratio was in good agreement with those calculated from the formulas in Figures 5A, 5B, 5C, and 5D. On the ray tracing analysis, there were small differences in subjects 3 and 4. The reason for these small differences is not known but the estimated figures on the position of the centre of the eye were considered to be the reason for the differences. In spite of the small differences, in general, the results of ray tracing were in fairly good agreement with the results of the geometrical analysis. These mathematical results agreed closely with the real measurement of the position of the corneal reflex through spectacles. It was clearly shown by these results that the effect of spectacles on the corneal reflex can be explained by the magnification effect of the lens.

In the clinical case, the angle λ is not 0° and the centre of the pupil, the centre of the corneal curvature, and the centre of rotation usually deviate from the same line. In our ray tracing analysis, when the optical centre of lens deviates from the centre of the pupil by 10 mm, the error is calculated to be less than 5% through lenses within +12D to -12D. As we have shown by the real measurement of the position of the corneal reflex, the real distances of the corneal reflex through spectacles can be estimated from the magnification factor. Of course, the easiest way to determine the magnification factor is by comparison of the corneal diameter without and with spectacles.

The CRT is frequently used in the short form as 15°, 30°, 45° as in Figure 6. As the corneal reflex, the cornea, and the pupil were magnified

in the same manner by spectacles around the optical axis of the lens, the short form of the CRT can be considered clinically useful even when spectacles are worn.

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