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PERISCOPIC LENSES

BY

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THE object of periscopic lenses is to enable the patient to see distinctly when ranging his eyes from side to side instead of being obliged to move his head, as is necessary when strong convex glasses are used. The problem of designing such lenses has been attacked by many workers for more than thirty years.

In 1898 Ostwalt did some excellent work which first directed my attention to the subject; three years afterwards I published a preliminary paper in Knapp's *Archives.* In 1908 Tscherning showed that for a certain range of convex glasses the astigmatism due to oblique vision through them can be entirely obviated, and that whenever this is the case, there are two forms of meniscus that will serve the purpose; one, a deep meniscus (Wollaston's), the other, a shallower form (Ostwalt's); he also published an elliptical curve from which these two forms of meniscus for any power within this range could be easily read off. Gullstrand about the same time showed that by means of *aspherical* surfaces the range of powers that could be entirely freed from astigmatism was greatly increased. These Katral lenses have to be worked by hand and are enormously expensive. In 1908 Tscherning published some tables, both for distance and for close work (*Receuil d'Ophtalmologie*) of all the powers 1 to 20D., both convex and concave, both periscopic and orthoscopic, that would best attain his object. The mathematical method he used (based on Czapski's formulae) is described in detail in the last edition of the *Encyclopédie d'Ophtalmologie*. We are all deeply indebted to Professor Tscherning for this magnificent work, but I think our master has made a slip. His orthoscopic lenses have been calculated with the greatest care to correct as far as possible the distortion and the curvature in the peripheral parts of the field on eccentric vision. But I contend that this is unnecessary. The eye is not like a photographic plate, equally sensitive all over; it is only the macula that can distinguish minute details of form. M. Dor has taught us that 5° from the fixation point the visual acuteness has diminished to 0.25 and at 10° to 0.066 of that at the macula. If the eyes be fixed on one letter of this Journal it will be found that the fourth letter on either side cannot be distinguished. Surely to the eye it is a matter of entire indifference whether the peripheral parts of the field are accurately focussed or not, they cannot be distinguished in any case.

We have all received advertisements of Punctal and Katral lenses with photographs of test types to show the advantage gained by these lenses in the definition of the peripheral parts of the field. I grant that they are far better for photographic purposes, but I maintain that such pictures are of no evidence to show that they are useful for visual purposes. The optical conditions are entirely different. The photographic lens is provided with a *fixed central* stop; the periscopic lens is provided with a *movable* stop (the pupil), and so gives a well defined macular image of the eccentric part of the field under examination when the eye ranges to one side. The macula is far more sensitive than any photographic plate, even to a thousandth of a millimetre, but at a short distance from the fovea all this extraordinary sensitiveness is lost. However, as the mathematical method involved in the calculation of these Punctal and Katral lenses has not been published (it is I believe a patent process), they belong to the class of secret remedies that cannot be here reviewed.

Recently Mr. Whitwell has published in the *Optician* some work that is based on Gleichen's formulae; he has also dealt with the subject of prescribing sphero-cylinders in a periscopic form. I must mention also a valuable paper of his in the same periodical on the important part that binocular vision plays in the apparent curvature of the field.

It will be well to take an extreme instance to show the nature of the defects in macular vision that arise from the ordinary biconvex lens. Suppose that an aphakic is given a +14 D. for reading at a distance of 30 cm. from the plane of his glasses. If he rotates his eyes 30°, there will be formed by this peripheral part of his reading glass two focal lines, the first (v′) at a distance of −39.02 mm., and the second (v″) at a distance of −65.65 mm. behind the glass.

Now $D' = \frac{v' - p}{v'p}$ in metres.

Here p is the distance of the object, but it is not now 0.3 m. for it is the slant distance at 30°, say 0.3 sec. 30°m. or 0.34641 m. (This is not exact, for we are neglecting the prismatic effect of viewing an object through an eccentric part of the lens.)

$$\text{Then } D' = \frac{-0.03902 - 0.34641}{(-0.03902)(0.34641)} = \frac{-0.38543}{-0.0135169} = +28.515\text{D}.$$

Similarly in the other meridian for the second focal line (v″)

$$D'' = \frac{-0.06565 - 0.34641}{(-0.06565)\ (0.34641)} = \frac{-0.41206}{-0.0277418} = +18.119$$

The astigmatism of this biconvex lens when viewed eccentrically at an angle of 30° is D′ − D,″ that is 10.395 D.

Further, we must find what should be the effective power of this lens when thus viewed eccentrically by an aphakic. As +14D. is the correction for centric vision at a distance of 0.3 m., the image must be formed at q, where

$$q = \frac{p}{1 - pD.} = \frac{0.3}{1 - 4.2} = -\ 0.09375\text{ m.}$$

Now if p for eccentric vision be regarded as 0.34641, we have

$$D_{1} = \frac{q - p}{qp} = \frac{-0.44016}{-0.0324476} = +\ 13.5534\text{D.}$$

Indeed, if we neglect the prismatic effect of the lens, whatever be the lens required for centric vision, for eccentric vision at this angle of 30°, its converging power should be made less by −0.4466D., for that is the lens that will form an image at 0.3 m. of an object at 0.34641 m.

Clearly then the biconvex lens not only has a huge astigmatic error, but its power when viewed eccentrically is far too great, so that the unfortunate patient would be obliged to move his head for almost each word that he reads.

The method that I have adopted is different I believe from that of all other workers on the subject in several details. In the first place the range of ***rotation of the eye is*** taken to be 30° in any direction from the primary position, it is not the extent of the ***field of vision***; indeed the field is rather greater for concave glasses and less for convex glasses. Then I have calculated the radius of the circle of least confusion given by the meniscus when the eye is directed 30° from the middle line, and I then find *r*, the radius of its retinal image (given in the Tables for convex lenses), using Tscherning's last constants for the eye which may be found in his

Physiologic Optics (1924). I endeavour to obtain a retinal image of this circle less than that of a macular cone, the radius of which is 0.001 mm. I give the residual astigmatism uncorrected in the column marked As., found as in the above instance with the +10D. biconvex; and I pay due regard in the case of reading glasses to the back focal distance of the lens from an object that is viewed eccentrically. As shown above the power of a convex reading glass should be about 0.5D. less for extreme eccentric vision, while that of a concave reading glass should be about 0.5D. greater (*i.e.*, more concave) than that required for centric vision.

This is a point that seems to have been neglected by most other workers on the subject. My method was, I hope, clearly explained in the July number of ***The Ophthalmoscope***, 1914, although there I did not allow for the thickness (t) of the lens in estimating its effective power, and the calculation was made for an angle of 25° instead of 30° as in the following table. As indicated above, my attention has been wholly concentrated upon the macular vision, and I have given the ocular curvature in dioptres which can be ground with the ***ordinary tools*** in use. My Tables differ very widely from Professor Tscherning's who uses a deeper form of meniscus for which in nearly every case a special tool will have to be made. Without any doubt this gives far better photographs, but as I have explained I do not think that is any evidence of superiority as spectacle lenses.

I have taken 3.2 mm. as the average diameter of the pupil in the iris plane (practically about 3.5 mm. in the first principal plane of the eye), and μ to be 1.523, which is the average British (and the standard American) value for the refractive index of spectacle glass. It would be quite possible to give an ocular curvature which would make this residual astigmatism vanish for powers up to + 7D., but often at the expense of making the eccentric power of the lens so faulty as to be practically useless; in addition the curvatures of each surface must be given to four decimal places, for which there are at present no available tools. It will be found that with my ocular curvatures r (the radius of the retinal confusion circle) is less than 0.001 mm. up to + 7D. for distance, and indeed up to + 8.5D. for close work. Beyond these powers completely satisfactory results could only be attained by means of the hand-made aspherical surfaces originated by Gullstrand.

However, the forms given up to + 14D. give the best results with ordinary spherical curvatures, and are quite satisfactory for smaller angles than 30°. It will be seen that by giving the + 14D. an ocular curvature of — 4.5D. the astigmatism is reduced from 10D. with a biconvex lens to less than 1.5D. for oblique vision at 30° and the eccentric power at this angle is between + 14.71D. and + 13.25D. I have taken my very worst example in the second

Table, but such a lens for reading is of enormous advantage to an aphakic. If horizontal astigmatism is present, as is usually the case, it can be easily corrected by tilting the lens.

TILTING SPHERICAL 10D. LENS μ=1.523.

Obliquity.	Spherical.	Cylindrical.
15°	10.23	0.723
20°	10.38	1.3746
25°	10.64	2.3132
30°	10.95	3.6488

The cylindrical effect of tilting a + 14D. lens 20° is 1.4 × 1.3746 or adding a convex cylinder of 1.924D. with its axis horizontal. The astigmatism tends to disappear in these operation cases, and it is only required to bend the legs of the spectacles upwards to lessen the cylindrical effect.

A solid angle of 60° is a very generous allowance; the printed line of this Journal at 32.7 cm. from the centre of rotation (or 30 cm. from the plane of the glasses) only requires a range of movement less than 10° to either side of the middle line, *i.e.*, less than a solid angle of 20°.

In the case of reading glasses, as indicated above, adjustment has been made as far as possible for viewing a flat object. It will be noted that periscopic lenses for near work are entirely different from those for distance. This is a point that is apparently unrecognized by the Jena School of Optics, and as far as I know Tscherning is the only previous publisher of such a table. In Henker's Theory of Spectacles, published by the Jena School, no indication of the curvatures of even the distance lenses is given, but on page 129, we learn to our surprise that their periscopic lens of + 8D. gives an astigmatism of 1.83D. "on refraction at the periphery," which I suppose means at an angle of 30°, as that is the extreme angle mentioned when dealing with the subject of periscopic vision. On the next page we find that their Punctal lens of + 8D. gives only an astigmatism of 0.05D. My reading periscopic compares very favourably with their Punctal lens, although my distance glass made with ordinary tools is not so good.

In order to show these lenses in a comparable manner the values given for *r* have been calculated in the same way for each case as for normal eyes; they are not true for aphakic eyes. In aphakia the size of the pupil varies very greatly in different cases, so that no exact determination of *r* can be given; it may be assumed that usually in aphakia *r* will be about half as great again as the value given in the Table.

We have all had patients who complained of the curvature of the field when high periscopic lenses were used; in most if not in all such cases it will be found that the lenses have too high a power. For instance for a + 10D. lens on a − 7D. base, the anterior curvature will be found to be + 17D. But such a lens has an *effective* power of + 18D. owing to the displacement forwards of the second principal point. The true formula for the anterior surface I gave in this Journal in 1924 (page 372) $d_1 = \dfrac{\mu.(D-d_2)}{\mu + t\ (D-d_2)}$ and is therefore + 16.1. I maintain now that the common complaint of curvature of the field is due to this cause, it has nothing to do with the real curvature of the image field, for in all my recent cases attention to this point has relieved the patients of all their symptoms. For this reason the value of *t*, the axial thickness of the lens, is a very important matter and it must not be less than that indicated in the Table, at any rate up to +7D. As the higher powers are not truly periscopic up to an angle of 30°, they may be made smaller and thinner as they will then be lighter. The method of determining *t* is explained below in the case of a + 10D. plano-convex lens or indeed of a lens of that power bounded by spherical surfaces of any curvature.

A periscopic lens will be of somewhat large size, say 40 mm. in its greater diameter (an 00 eye as it is called); if then such a disc be ground to a convexity of +10D. (*i.e.*, a curvature of radius 52.3 mm.), until the edge is indefinitely thin, its axial thickness will be r $(1-\cos\theta)$. But we do not know cos θ, we only know sin θ which is $\frac{p}{r}$, where p is one half the diameter of the lens or 20 mm.

$$\text{So } r - \cos\Theta = r - r\sqrt{1-\sin^2\Theta} = r - \sqrt{r^2 p^2}.$$

$$\text{and } 52.3 - \sqrt{2735.29 - 400} = 52.3 - 48.32 = 3.98 \text{ mm.}$$

As the lens must be at least 1 mm. thick at the edge we add 1 mm. to this value, and obtain 4.98 mm. as the minimum thickness *t* of this +10D. lens.

Practically a fairly good estimate of the thickness of a lens of any power D. and of width 40 mm. can be obtained by using this empirical rule:—

$$t = 1 + 0.4D. = 5 \text{ mm. if } D. = 10$$

The distance of the lens from the eye must also be carefully considered, as a small variation in this will destroy the full periscopic effect of the most perfectly designed meniscus. My calculations are based on the assumption that the distance (BM.) from the ocular surface of the meniscus to the "centre of motility" (M.) of the eye is in every case 27 mm. The usual distance of M. from the apex of the cornea seems to be 13.4 mm., so that allows a distance of 13.6 mm. between the cornea and the correcting glass which will be quite enough to avoid contact with even long eyelashes. The distance allowed for this purpose is variable, Henker gives 12 mm. as the distance accepted at Jena, this I feel is not enough for most patients, at the same time I think that the usual allowance of 15 mm. is unnecessarily great.

We have found that for a + 10D. lens, when μ is 1.523, t is 4.98 mm., or 0.00498 m., for in all dioptric formulae the metre must be taken as the unit. D is the *effective* power of the lens, such that its second principal focus will be formed at a distance of 100 mm. from its posterior surface, as in the case of a+10D. trial lens placed 13.6 mm. in front of the cornea. In this case $d_2 = -7$, so $D - d_2 = 10 + 7 = 17$, and according to the formula

$$d_1 = \frac{17 \times 1.523}{1.523 + 17 \times 0.00498} = 16.104773D.$$

The *real* power of this meniscus (+ 16.104773 D. and − 7 D. where t = 4.98 mm.) is 9.4734D., and its focus is formed at 105.5587 mm. behind the second principal point, which is 5.5587 mm. in front of the ocular surface of the lens. The *effective* power of the lens for the correction of refractive errors is therefore exactly that of the trial +10D. lens placed in the correct position before the eye. The *real* power (+ 9.4734D.) of this meniscus we have to consider when we are dealing with the size of the image and other problems. It will be noted that the values of d_1 in the Tables are only given to two decimal places, as they only indicate what should be aimed at; it is always better to prescribe an anterior curvature less rather than greater than the stated value of d_1. The correct form of a +10D. lens for distance is $\frac{+16D}{-7D.}$ which will be quite satisfactory to the patient.

Now if a periscopic sphero-cylinder is required, we encounter great difficulties, as toric lenses are only made on a basis of 3D., 6D. and 9D., so that unless special tools are made for the purpose some thought must be given to the matter before the best attainable correction can be prescribed. It is usually better to make the anterior surface a convex toric, and the ocular surface a spherical concave; otherwise with a concave toric, if the cylinder be fairly high, although in one meridian the curvature may be appropriate, it will be hopelessly wrong in the other: it will be noticed in the Table that the convexity of the anterior surface increases much more rapidly than the change of concavity of the ocular surface. Further, I would point out that the change in the concave surface is not uniform, the concavity first increases and then diminishes, this change occurs when there is a change of sign in the incident angle.

Suppose that + 2D. sph. + 3D. cyl. ax. 90° is required in periscopic form for reading. The Table shows that between + 2D. and + 6D. the ocular curvature should be between − 4.25D. and − 4.5D., so an ocular curvature of $-$ 4.25D. should give a very good result, and we may write tentatively this prescription, subject to further testing.

$$\frac{+\ 6\text{D. cyl. ax. } 0^\circ\ \textit{cum} + 9\text{D. cyl. ax. } 90^\circ}{-\ 4.25\text{D.}} \quad \text{Toric surface anterior.}$$

Testing the prescription. From the Table it is seen that in order to obtain a power of + 2D. on a concave surface of $-$ 4.25D. with a thickness of 1.77 mm. the anterior curvature should be + 6.2D., but this lens must have the thickness of a + 5D. glass. By the rule for t previously given

$$t = 1 + 0.4\text{D.} = 1 + 2 = 3 \text{ mm.}$$

It is required to find the *effective* power D. of this meniscus in the two meridians. By a simple transposition of the formula given on page 374 it is seen that $\mu\ (\text{D} - d_2) = d_1\ \mu + d_1\ t\ (\text{D} - d_2)$

or $\text{D} - d_2 = \dfrac{d_1\ \mu}{\mu - d_1\ t}$

(1) In the horizontal meridian $\text{D}' - d_2 = \dfrac{9\mu}{\mu - 0.027} = \dfrac{13.707}{1.496} = 9.16$

$$\therefore\ \text{D}' = -4.25 + 9.16 = 4.91$$

(2) In the vertical meridian $\text{D}'' - d_2 = \dfrac{6\mu}{\mu - 0.018} = \dfrac{9.138}{1.505} = 6.07$

$$\therefore\ \text{D}'' = -4.25 + 6.07 = 1.82$$

The effective value of this periscopic lens is therefore

+ 1·82 D. + 3.09 D. cyl. ax. 90.°

The spherical correction is 0.18 too small (which is in the right direction), and the cylinder is 0.09 too great. This will be the right addition to make to a + 3 D. cyl., as found for distance, if required for reading and + 2 D. has been given for the presbyopic correction; as explained in my paper on the "Alteration of the power of a cylinder when used for close work" in this Journal last year (page 8). The prescription can then be given with full confidence.

Now if the same prescription + 2 D. + 3 D. cyl. ax. 90° were required in periscopic form for distance, it is a far more troublesome matter. From the Table the ocular curvature should be that of $-$ 6.5 D., but no suitable toric glass such as + 8.36 cyl. ax. 0° *cum* + 11.36 cyl. ax. 90° can be supplied without having special tools made for the purpose.

If we are limited to the available toric surfaces we must consider the relative merits of:

(1) An anterior toric of +9 D. cyl. ax. 0° *cum* + 12 D. cyl. ax. 90°.
(2) An ocular toric of $-$6 D. cyl. ax. 90° *cum* $-$ 9 D. cyl. ax. 0°.
(3) An ocular toric of $-$ 3 D. cyl. ax. 90° *cum* $-$ 6 D. cyl. ax. 0°.

At the first glance remembering that the ranging movements of the eyes are chiefly in the horizontal direction one would be inclined to choose (2) as it will give the best curvature $-$ 6 D. in this meridian. This is true, but it will not correct the astigmatism when ranging the eyes in *all* directions as is so necessary in sport and outdoor games.

Certainly (1), bad though it is, will give the best result for *all* lines with a limited range of movement; at any rate it will be far better than the ordinary sphero-cylinders for such as lawn tennis players who require periscopic vision in all directions. This consideration will show the very common error of some opticians in making periscopic lenses always on a toric base of -6 D.

(For reading glasses a toric ocular surface would be allowable, although for really accurate vision the head must be depressed for every three or four lines, while for ranging movements of the eyes from side to side this ocular toric arrangement would serve very well. But unfortunately torics are not made in any of the powers required from -4.25 to -5.50.)

We must find the best value to give to the ocular curvature (d_2) by using the same formula as before $D. - d_2 = \dfrac{d_1 \mu}{\mu - d_1 t}$

$$(1)\ D' = 2 \text{ and } d' = 9,\ (2)\ D'' = 5 \text{ and } d_1'' = 12$$

$$(1)\ 2 - d_2' = \frac{9\,\mu}{\mu - 0.027} = \frac{13.707}{1.496} = 9.16$$

$$(2)\ 5 - d_2'' = \frac{12\,\mu}{\mu - 0.036} = \frac{18.276}{1.487} = 12.29$$

$$\therefore\ d' = 2 - 9.16 = -7.16, \text{ and } d_2'' = 5 - 12.29 = -7.29$$

The prescription $\dfrac{+\ 9D.\ \text{cyl. ax. } 0^\circ\ \textit{cum}\ +\ 12D.\ \text{cyl. ax. } 90^\circ}{-7.25D.\ \text{sph.}}$

is then the best obtainable with the stock tools in the workshops.

We may then reasonably order the ocular curvature to be -7.25D. It is, however far from perfect, the effective power of this meniscus will be $+1.91$D. sph. *cum* + 3.13D. cyl. ax. 90°, and owing to the ocular curve being -7.25D., the result will not be periscopic for a range of 30° on either side of the middle line, though probably for a range well over 20°.

From these examples it is hoped that the procedure in dealing with periscopic sphero-cylinders will be now quite clear to those who have a taste for such problems. I have said enough to show that it is not fair to expect the busy optician to spend the time and thought required for them. Ophthalmic surgeons who have no taste for these things I strongly recommend not to order periscopic sphero-cylinders; to some however there is a special interest in working out the best possible meniscus with the available tools.

TABLE OF CONVEX PERISCOPIC LENSES FOR DISTANCE

Solid Angle of 60° ; $\mu = 1.523$

D	t	Ant. Surf.	Ocular Surf.	As.	r
+ 2	1.77	+ 8.42	− 6.5	0.00210	0.0000327
+ 4	2.54	+ 10.32	− 6.5	0.00857	0.0001374
+ 6	3.33	+ 12.64	− 7.0	0.01510	0.0002504
+ 8	4.14	+ 14.41	− 7.0	0.14956	0.002576
+ 10	4.98	+ 16.10	− 7.0	0.42782	0.007681
+ 12	5.86	+ 17.49	− 6.75	0.90829	0.017103
+ 14	6.79	+ 18.15	− 5.75	1.57969	0.03133

TABLE OF CONVEX PERISCOPIC LENSES

Distance 30 cm. from Spectacle Plane

D	t	Ant. Surf.	Ocular Surf.	As.	r
+ 2	1.77	+ 6.21	− 4.25	0.001602	0.00002433
+ 4	2.54	+ 8.14	− 4.25	0.00323	0.00005216
+ 6	3.33	+ 10.26	− 4.50	0.00499	0.0000784
+ 8	4.14	+ 13.02	− 5.50	0.01627	0.0002651
+ 10	4.98	+ 14.30	− 5.00	0.27045	0.0045496
+ 12	5.86	+ 15.96	− 5.00	0.7535	0.01327
+ 14	6.79	+ 17.09	− 4.5	1.4475	0.02679

It only remains to add a few words about concave periscopic lenses. Spherical concave lenses are much more amenable to periscopic treatment, as they can be made entirely satisfactory up to powers over −20D. with a minimum amount of residual astigmatism and a tiny retinal confusion circle smaller than a macular cone. For this reason I have omitted the two columns for r and As. I have also omitted the column for t as the axial thickness is always 1 mm. There is no necessity to calculate the anterior curvature, for as will be seen there is only occasionally an alteration in the second decimal place of its anterior dioptric power. There is however an even greater difficulty in dealing with sphero-cylinders in this case, for it will be noted from the table that with each change of power there is a change in the ocular as well as in the anterior curvature. It will make little difference whether we make the anterior or the posterior surface toric, the result will not be fully periscopic in either case; for an accurate periscopic sphero-cylinder it would be necessary to have both anterior and ocular surfaces toric and for each of these surfaces special tools would have to be made. In some cases of high myopia and low astigmatism with the axis of the cylinder horizontal a satisfactory result may be obtained by ordering the simple spherical from the table and giving them the appropriate tilt to correct the astigmatism. However if the ordinary prescription in high myopia be written with the cylindrical surface anterior a fairly good periscopic result will be often obtained, as good or better than can be obtained with one toric surface.

The optical results in simple myopia are excellent, but in high myopia it should be remembered that when looking downwards ranging movements and convergence of the eyes should be discouraged.

TABLE OF CONCAVE PERISCOPIC LENSES

Solid Angle of 60°. $\mu=1.523$

D	Distance		Distance 30 cm.	
	Ant. Surf.	Ocular Surf.	Ant. Surf.	Ocular Surf.
− 2	+ 5.73	− 7.75	+ 3.74	− 5.75
− 4	+ 4.74	− 8.75	+ 2.50	− 6.50
− 6	+ 3.74	− 9.75	+ 1.50	− 7.50
− 8	+ 2.75	− 10.75	+ 0.50	− 8.50
− 10	+ 2.00	− 12.00	− 0.50	− 9.50
− 12	+ 1.25	− 13.25	− 1.25	− 10.75
− 14	+ 0.75	− 14.75	− 2.00	− 12.00