CO-OPERATIVE ACTION OF EXTRA-OCULAR MUSCLES*

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IN every excursion of the eyes, two times six muscles act with extraordinary speed, precision, and harmony in rotating them into a new position of fixation. Some of these muscles contract while the others extend, and this is achieved in countless variations and apportionments.

In a previous study (Boeder, 1961), the co-operative action of the extra-ocular muscles was analysed for several types of ocular rotations. With the measurements given by Volkmann (1869) of the locations of the effective origins and insertions of the six extra-ocular muscles as a basis, the simultaneous changes in length which take place in these muscles in these rotations were calculated on the assumption that there is no muscle slack in any equilibrium position of the eye, and that for any fixation consistent with Listing's law, there exists one and only one set of contractive states of the muscles, no matter from which direction the fixation is reached.

The purpose of the present study is to extend this analysis to the general case, that is, to any conceivable ocular rotation in a $60^\circ \times 60^\circ$ field of fixation, to determine the component directions of pull, and to present the results in six easily readable charts.

To accomplish this we proceed as follows:

Ocular fixations and rotations are specified by marking the points where the line of fixation of the mobile eye pierces a concentric stationary reference sphere of a radius equal to that of Volkmann's eye (12.25 mm.). There is, for instance, a point on the reference sphere which marks the primary position of the eye. Any other normal fixation can be regarded as having been reached from the primary position by a rotation about an axis lying in Listing's plane. When the eye rotates about such an axis from the primary position to any other, the line of fixation traces an arc of a great circle on the reference sphere. At the same time, the insertion of any extra-ocular muscle travels on a parallel circle from its "primary position" through an arc which subtends the same angle as that described by the line of fixation. This establishes a correspondence between a normal fixation and the position (as well as the orientation) of the muscle's insertion; that is, for any point in the

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field of fixation there is a corresponding point in the "field" of the muscle's insertion, and *vice versa*.

Moreover, if such points in the field of insertion lie on a certain curve, the corresponding points in the field of fixation also lie on a curve, called the corresponding curve.

Let the straight line through the muscle's origin and the centre of rotation of the eye be the axis of a set of parallel circles on the reference sphere. One of these circles goes through the primary position of the muscle's insertion. Let the number 0 be assigned to this circle, because if the insertion happens to be on this circle, the muscle is the same length as when the eye is in the primary position. The set of parallel circles also contains circles that are 1, 2, 3 mm., etc., nearer to the muscle's origin (the distance measured on the sphere), and others that are 1, 2, 3 mm., etc., farther from it. We designate these circles -1, -2, -3, etc., and 1, 2, 3, etc., respectively. Thus, if the insertion happens to be on circle -2, the muscle is 2 mm. shorter than in the primary position; and if the insertion happens to be on circle 3, the muscle is 3 mm. longer than in the primary position.

These numbered circles in the field of insertion have corresponding curves in the field of fixation, to which we give the same numbers. Thus, if the line of fixation happens to be on curve -4, the muscle is 4 mm. shorter than in the primary position, and when the line of fixation happens to be on curve 1, the muscle is 1 mm. longer than in the primary position.

Next let us consider, in the field of insertion, a set of great circles that intersect the set of numbered parallel circles at right angles. The important property of these great circles is that they represent the lines of pull of the individual muscle; that is, if the muscle could act alone, the insertion would travel along such a great circle. Under this condition, the eye would rotate about the axis perpendicular to the plane of the great circle.

Each of these great circles in the field of insertion has, of course, a corresponding curve in the field of fixation. If we could identify the particular corresponding curve on which the line of fixation happens to be at a given moment, we should know the corresponding great circle in the field of insertion and consequently also the axis about which the eye would rotate, if the muscle acted alone. The line of fixation, rotating with the eye, would then trace an arc of a circle on the reference sphere.

If we were interested only in the direction in which the line of fixation at a given point would be pulled by the muscle acting alone, we should not consider the entire circular arc that would be traced by the line of fixation, but only the tangent to it at the given initial point. In this way, we could associate with every point in the field of fixation a direction in which the line of fixation, when occupying this point, would be pulled if the muscle acted alone. The envelopes of these directions (*i.e.* the curves to which these directions are tangent) form a set of curves which we shall call direction curves.

Mathematically, the curve in the field of fixation that corresponds to a given curve in the field of insertion is found as follows:

As pointed out previously, the great circles which are traced in the field of fixation when the eye rotates from the primary position to any other, have corresponding parallel circles through the primary position of the muscle's insertion. It is necessary to determine the relevant intersections of some of these circles with the given curve in the field of insertion, as well as their angular distances, measured at the centre of the respective circles, from the primary position of the insertion. In this way, an arc on a specific circle through the primary position of the insertion is associated with every intersection. The points that correspond to these intersections are then found in the field of fixation by tracing from the primary position arcs of the same angular measurement on the corresponding great circles. The points thus determined constitute the curve corresponding to the given curve.

As an example, let circle 0 be given in the field of insertion. Let the rectangular co-ordinate system x_1, x_2, x_3 , be such that its origin coincides with the centre of rotation of the eye, and the x_1 axis with the line of fixation in the primary position. Then the great circles through $r, 0, 0$, of the reference sphere

$$x_1^2 + x_2^2 + x_3^2 = r^2 \quad (1)$$

take the form

$$a_2x_2 + a_3x_3 = 0,$$

and the parallel circles through the insertion, i_1, i_2, i_3

$$\text{are} \quad a_2x_2 + a_3x_3 = a_2i_2 + a_3i_3. \quad (2)$$

The 0-circle with points equidistant from o_1, o_2, o_3 , the origin of the muscle under consideration, is given by

$$o_1x_1 + o_2x_2 + o_3x_3 = o_1i_1 + o_2i_2 + o_3i_3. \quad (3)$$

The solutions of equations (1), (2), (3) yield the coordinates of the intersections.

Let such an intersection be b_1, b_2, b_3 . Since the centre of the specific circle must satisfy (2) and also the straight line $x_1=0, x_2=a_2t, x_3=a_3t$, its coordinates are readily found; let them be c_1, c_2, c_3 . Then, α , the angular distance of the intersection from the primary position of the insertion, is obtained from:

$$\sin^2 \frac{\alpha}{2} = \frac{(b_1 - i_1)^2 + (b_2 - i_2)^2 + (b_3 - i_3)^2}{(b_1 - c_1)^2 + (b_2 - c_2)^2 + (b_3 - c_3)^2}.$$

Finally, the point in the field of fixation that corresponds to b_1, b_2, b_3 , is located on the corresponding great circle at an angular distance, α , from the primary position.

Similarly, we find, in the field of fixation, the curves that correspond to the circles $\pm 1, \pm 2$, etc., in the field of the insertion.

If the given curve is one of the great circles that are perpendicular to the numbered circles, we have instead of (3),

$$d_1x_1 + d_2x_2 + d_3x_3 = 0, \quad (4)$$

where d_1, d_2, d_3 , are functions of o_1, o_2, o_3 , and the coordinates of an arbitrarily chosen point on the circle.

From the curves that correspond to the great circles (4), we then determine by means of graphical integration, a sufficient number of direction curves whose tangents, as explained previously, indicate the directions in which the line of fixation would be pulled if the muscle in question acted alone.

In this manner, the curves marking the state of muscular contraction, as well as the direction curves, are obtained for all extra-ocular muscles, and presented in six charts (Figs 1-6, opposite), each of which represents the field of fixation of the right eye, ranging from 30° of abduction to 30° of adduction, and from 30° of depression to 30° of elevation.

In all charts, the heavy numbered lines are the loci of those normal fixations for which the muscle in question is contracted as many millimetres as their negative numbers indicate. The broken numbered lines are the loci of those normal fixations for which the muscle is extended as many millimetres as their positive numbers indicate. The curves which cross these curves mark the direction in which the line of fixation, when occupying one of their points, would be pulled, if the contracting muscle acted alone at this point.

The charts show only a few curves of either set. For fixations not located on the curves shown, the state of muscular contraction and the direction of pull can be estimated by interpolation.

What can be learned from these six charts? A few examples follow:

(1) For any fixation in the region considered, the contractive state of all six muscles can be determined. For instance, the contractive state of all muscles (in mm.) at the fixation 20° in and 30° up is as follows:

L.R. 2, M.R. -4.7, S.R. -6.2, I.O. -4.3, I.R. 3.6, S.O. 5.3;

whereas, at the fixation 20° in and no elevation it is:

L.R. 4.4, M.R. -4.3, S.R. -1.2, I.O. -1.3, I.R. -1.3, S.O. 0.5.

(2) For an excursion from one fixation to another, the change in contractive state can be determined by taking the difference between the two states corresponding to the two fixations. For instance, in the excursion from 20° adduction and no elevation to 20° adduction and 30° elevation, the change in contractive state is:

L.R. -2.4, M.R. -0.4, S.R. -5, I.O. -3, I.R. 4.9, S.O. 4.8.

Four muscles have contracted (L.R., M.R., S.R., I.O.), and two (I.R. and S.O.) have extended.

This change in contractive state represents the minimal muscular requirement for an excursion between those two fixations. There are, of course, more devious routes along which muscular changes occur that are cancelled before the eye arrives at the final fixation. If we are interested in the total required changes along such a route, it must be divided into sub-routes, along each of which the contractive state of each muscle either never decreases or never increases; because the difference between the contractive states of two fixations represents the minimal muscular requirement for an excursion between them only if a route exists along which each muscle changes monotonically. With this proviso, it is a simple matter to determine by inspection of the charts, the qualitative and quantitative action of the muscles along any chosen path.

FIG. 1.

FIG. 2.

FIG. 3.

FIG. 4.

FIG. 5.

FIG. 6.

(3) When this has been done for an excursion from A to B, we may ask how the return excursion from B to A is accomplished. In accordance with our assumption of a unique contractive state of the extra-ocular muscles for every normal position of the line of fixation, we find that the muscular action is exactly reversed; the muscles that contracted in the excursion from A to B, extend again in the return excursion and *vice versa*. Concerning the muscles that contract in the excursion from A to B and those that contract in the return excursion, we find that their number is usually different, and their directions of pull are by no means always opposite. It follows that the direction of an excursion is not determined by the contracting muscles alone; if it were, the excursion from A to B would in general be irreversible. We conclude that the direction of an excursion is determined not only by the contracting muscles but also by the extending muscles, and that, discounting inaccuracies of performance, the directional influence of an extending muscle is equal in absolute magnitude, but opposite in direction to that which the muscle would have as a contractor in the counter-excursion.

For instance, contemplating pure elevation from the primary position, we may ask, since Figs 3 and 4 show adducting components for both elevators, why the eye does not slightly adduct, at least in the beginning of the elevation. The answer is, that these adducting components are cancelled by the abducting directional component of the extending inferior rectus. This component also cancels the adducting directional component of the extending superior oblique, and, to be exact, is aided in this by a small differential abducting component of the contracting horizontal recti.

As another example, we may once more consider elevation from a position of adduction. We found previously that, in elevating the eye from a horizontal position of adduction, not only the elevators but also the horizontal recti contract, especially the lateral rectus. From Fig. 1 we find further that, in elevation, the lateral rectus contracts increasingly with increasing adduction of the starting position. This contraction takes place in what is usually not regarded as the "field of action" of the lateral rectus; rather, it is a reduction of extension. We may wonder, therefore, whether the lateral rectus merely takes up slack or actually contributes to those elevations. Referring to Figs 3 and 4, we see that the directions of pull of the elevators for starting positions of increasing adduction have increasing components of adduction. Consequently, if the lateral rectus did not actively co-operate, the eye would not only be elevated, but at the same time adducted. We are forced to conclude that the elevators cannot elevate the eye from positions of adduction without the lateral rectus; the latter's abducting component must help the extending inferior rectus in cancelling their adducting components.

Inspection of Figs 1, 5, and 6 reveals that the lateral rectus has a corresponding role in depression; and Figs 2 to 6 show that the medial rectus co-operates similarly in elevation and depression when the eye starts from positions of abduction.

(4) Figs 3 to 6 show that, in vertical excursions to the 30°-border of the chosen fixation range, the superior and inferior recti contract, on the average, about 6 mm., whereas the inferior and superior obliques contract only about 4.5 mm. and 3 mm. respectively. This confirms the previous analytical finding, namely, that the superior rectus contributes to elevation more than the inferior oblique, even when

the eye is simultaneously adducted; and similarly, that the inferior rectus contributes more to depression than the superior oblique throughout the entire considered range of abduction and adduction. Thus, the generally accepted view that the action of the vertical muscles is divided between the recti and the obliques, so that the recti act primarily in abduction and the obliques primarily in adduction, is not supported by this analysis.

Since this view has provided the basis for the assumption that the vertical recti of one eye are yoked with the obliques of the other, this assumption, used in explaining ocular deviations in cases of vertical muscle defect, is also challenged. It is true, as Figs 3 to 6 show, that the vertical recti are more efficient in abduction than in adduction, whereas the obliques are more efficient in adduction than in abduction; but in the teamwork of the pair of elevators and the pair of depressors, the larger share is always done by the recti, never by the obliques. The analysis presented in the charts suggests that the assumed cross-yoking of individual vertical muscles does not exist, but that the pair of elevators of one eye is yoked, as a unit, to the pair of elevators of the other; and that the two pairs of depressors are similarly yoked.

The final proof can come only from experiment; thus, electromyography, applied to cases of paralysis of a single vertical muscle, could decide the issue.

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