Calculation of total retinal area

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The general topography and the linear measurements of the eye have long been accurately documented, but when, for a recent paper (Taylor, 1970), it was desirable to know the total retinal area, no reference for this was found. A calculation of total retinal area was therefore attempted, using mean linear and radial measurements for the emmetropic eye (Duke-Elder and Wybar, 1961; Thiel, 1955). It is not suggested that the result is exact but it seems the best available at the present time, using these data.

The following values were assumed in the calculation:

Outer sagittal diameter of globe	24.15 mm.
Inner sagittal diameter of globe	22.12 mm.
Diameter of optic disc	1.50 mm.

Distance of ora serrata from limbus in:

nasal meridian	6.60 mm.
superior meridian	7.40 mm.
temporal meridian	7.90 mm.
inferior meridian	6.90 mm.

Diameter of cornea:

vertically	10.60 mm.
horizontally	11.70 mm.

Figs 1 and 2 are schematic representations of the eye, Fig. 1 representing a narrow spherical lune and Fig. 2 a sagittal section through the schematic eye. The letters and angles indicated are used in the text.

Consider the right eye (Fig. 1)

Let PQ be the geometrical axis of the eye. Let O be the centre of the sphere. Let PSC and PS'Q be two great circles separated by a small angle, $\delta\xi$, and enclosing a narrow spherical lune.

The area of the shaded element of this lune is the product of length and breadth, equal to $\delta x \cdot \delta z$. If r is the inner radius of the eye and the small angles are measured in radians we have:

$$\begin{aligned}\delta x &= r\delta\xi \cdot \sin\zeta \\ \delta z &= r\delta\zeta.\end{aligned}$$

Therefore the area of the shaded element is $r^2\delta\xi \sin\zeta \delta\zeta$.

FIG. 1 *Diagram of a narrow spherical lune* FIG. 2 *Diagram of a sagittal section through the schematic eye*

- LL' = corneal diameter
- LQ' = $\frac{1}{2}$ LL'
- OL = R
- $\sin \theta$ = LQ'/R
- θ = $\sin^{-1}$ LQ'/R, the angle whose sine is LQ'/R
- LS = ora serrata to limbus
- ψ = LS/R (radians)

If the retina extends from P posteriorly to SS' anteriorly the retinal area enclosed by this lune is:

$$\int_0^{\text{POS}} r^2 \delta \zeta \sin \zeta \cdot d\zeta = r^2 \delta \zeta (1 + \cos \psi)$$

Therefore, if we could describe the angle ψ in terms of the angle ξ about the geometrical axis by a function $F(\xi)$ such that $\psi = F(\xi)$, we could calculate the retinal area as:

$$\int_0^{2\pi} r^2 (1 + \cos F(\xi)) \cdot d\xi,$$

from which we must subtract the area of the optic disc.

From the data of vertical and horizontal corneal diameters and of the distance of the ora serrata from the limbus in the four main meridians, it is possible to calculate the angle ψ at these meridians. Since the measurements given are external, in these calculations we make use of the outer sagittal radius, here denoted by R , instead of the inner, r . A reasonable assumption has to be made as to the shape of the ora serrata to give values of ψ at intermediate values of ξ which represent parts other than the four main meridians. This assumption is used in place of the expression of ψ as a formal function.

In Fig. 2.

LL'	=	corneal diameter (known)
LQ'	=	$\frac{1}{2}$ LL'
OL	=	R (known)
Sin θ	=	LQ'/ R
θ	=	$\sin^{-1}$ LQ'/ R , the angle whose sine is LQ'/ R .
LS	=	distance of ora serrata from limbus (known)
OL	=	OS = R
ϕ	=	LS/ R (in radians)

Hence we can calculate $\psi = \theta + \phi$ at each of the four main meridians. The Table shows the results of these calculations.

Table

Meridian	ξ	sin θ	θ	ϕ	ψ
Nasal	0	0.4845	0.5058 (28° 59')	0.5467 (31° 19')	1.0525 (60° 18')
Superior	$\pi/2$	0.4389	0.4544 (26° 02')	0.6128 (35° 07')	1.0672 (61° 09')
Temporal	π	0.4845	0.5058 (28° 59')	0.6543 (70° 29')	1.1601 (66° 28')
Inferior	$3\pi/2$	0.4389	0.4544 (26° 02')	0.5714 (32° 44')	1.0258 (58° 46')

It is reasonable to suppose that in each quadrant the angle ψ varies linearly with ξ ; that is, we assume that the retinal margin runs smoothly around the eye, the scalloping of the margin being neglected. This assumption implies that in the upper medial quadrant, between the nasal and superior meridians, ψ is given by:

$$\psi = \psi_{\text{nasal}} + \xi/\frac{1}{2}\pi (\psi_{\text{superior}} - \psi_{\text{nasal}}) = 1.0525 + 0.00936\xi$$

If we put $a = 1.0525$ and $b = 0.00936$, we find the area of the superior nasal quadrant to be:

$$\int_0^{\frac{1}{2}\pi} r^2 (1 + \cos(a + b\xi)) \cdot d\xi$$

$$\begin{aligned} &= \frac{\pi r^2}{2} + r^2 \left[\frac{\sin (a + b\zeta)}{b} \right]_0^{\frac{1}{2}\pi} \\ &= r^2 \left(\frac{\pi}{2} + \frac{\sin 1.0672 - \sin 1.0525}{0.00936} \right) \\ &= r^2 (1.5708 + 0.7746) = 286.8 \text{ sq.mm.} \end{aligned}$$

Similar calculations for the other quadrants give the following results:

Area of superior nasal quadrant	286.8 sq. mm.
Area of superior temporal quadrant	276.8 sq. mm.
Area of inferior temporal quadrant	280.4 sq. mm.
Area of inferior nasal quadrant	289.8 sq. mm.
Total	1133.8 sq. mm.

The disc area remains to be calculated and subtracted. It is given by multiplying the square of the disc radius by π . This amounts to 1.8 sq.mm. The subtraction gives the total area of the retina as 1132 sq.mm.

Obviously the decimal places have been retained only to guard against the accumulation of errors through rounding off.

We suggest that the retinal area be taken as 1100 sq.mm.

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References

DUKE-ELDER, S., and WYBAR, K. C. (1961) In "System of Ophthalmology", vol. 2, "The Anatomy of the Visual System", pp. 80, 93, 271, 287. Kimpton, London
TAYLOR, E. (1970) *Brit. J. Ophthal.*, **54**, 535
THIEL, H. L., (1955) *v. Graefes Arch. Ophthal.*, **156**, 590