PRESCRIBING SPECTACLES (2)

ALTERATION OF THE POWER OF A CYLINDER
WHEN USED FOR CLOSE WORK

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AFTER correcting a high degree of astigmatism and obtaining a macular visual acuteness of 6/6, or at any rate some of the letters of that line, I have often been disappointed with the result I obtained when testing the patient's near vision. I could not understand it, and five months ago I determined to see if the effective value of the cylinder was altered when used for close work. To my surprise and to my delight I found that this was so, and my difficulty is thereby explained; on increasing the power of the cylinder appropriately I have now obtained better visual results at reading distance.

Far Vision.—Parallel rays incident on a convex lens at F^1 (the anterior focal plane of the eye) come to a focus at f'' , the focal distance being $f'' F^1$. The focal distance of the equivalent contact lens placed at A (the cornea) will be $f'' A$ or $f'' F^1 + F^1 A$.

If the lens at F^1 be of power D , and that at A be of power D_1 ,

$$D_1 = \frac{-1}{f'' F^1 + F^1 A} = \frac{-1}{\frac{-1}{D} + l} = \frac{D}{1 - lD} \quad \dots \dots (1)$$

where l or $F^1 A = \cdot 01375\text{m}$.

I use Tscherning's constants for the eye which I find are far more accurate for British eyes than Gullstrand's.

Near Vision.—Suppose that the object viewed (P) be at the distance p from the plane of the spectacles, so that $p = PF^1$; the image of P is formed at a distance QF^1 or q from F^1 , and we have

$$q = \frac{pf''}{p - f'} = \frac{-\frac{p}{D}}{p - \frac{1}{D}} = \frac{p}{1 - pD}$$

If now D_1 be the equivalent contact lens placed at A, and $p_1 = p + l$, or $PF^1 + F^1A$, and $q_1 = q + l$ or $QF^1 + F^1A$, we have

$$D_1 \text{ or } \frac{1}{f'} = \frac{1}{p_1} - \frac{1}{q_1} = \frac{q_1 - p_1}{p_1 q_1} = \frac{(q+l) - (p+l)}{(p+l)(q+l)} = \frac{q-p}{(p+l)(q+l)}$$

Denote $p+l$ by a ; in the following tables I have taken p to be 30 cm. and so a is .31375 m.

$$L_1 = \frac{q-p}{a(q+l)} = \frac{\frac{p-p+p^2D}{1-pD}}{ap+al-alpD} = \frac{p^2D}{a(p+l)-alpD}$$

$$\therefore D_1 = \frac{p^2D}{a^2-alpD} = \frac{\frac{p^2}{D}}{\frac{a^2}{D}-alp} \dots\dots\dots (2)$$

From these two formulae the values of D^1 and D_1 can be readily determined, as given below for four different powers of both convex and concave lenses; of which use will be made hereafter.

	D^1	D_1		D^1	D_1
+ 4	+ 4.2328	+ 3.8601	- 4	- 3.7915	- 3.4744
+ 7	+ 7.7455	+ 7.0486	- 7	- 6.3854	- 5.8605
+ 9	+10.2710	+ 9.3323	- 9	- 8.0089	- 7.3578
+12	+14.3713	+13.0264	-12	-10.3004	- 9.4762

When we are dealing with distant vision it will be seen from the values of D^1 that convex lenses in the usual place (F^1) have the same effect as stronger lenses in contact with the cornea, in fact a + 4 D cylinder corrects a corneal astigmatism of nearly + 4.25D of astigmatism; but concave lenses are equivalent to weaker corneal contact lenses. If, however, we are considering convex lenses used for near vision (D_1) we find that a curious change takes place, in this instance when the distance of F^1 from the object (p) is 30cm., cylinders below +7D correct a corneal astigmatism of less than its nominal value, but cylinders above +7D correct a higher degree of corneal astigmatism. I will not now give the mathematical explanation of this fact.

The important point to notice is that in every case D_1 is less than the corresponding D^1 , and as the corneal astigmatism is always determined, whether by retinoscopy or by test types, for distant vision, it is obvious that in simple astigmatism if of high degree a stronger cylinder will be required for reading. If - 5D cyl.

is required for distance, a $-5.5D$ cyl. will be needed to correct this astigmatism for reading. If $+4D$ cyl. is the accurate correction for distance the corneal astigmatism is D^1 or 4.2328 , and we must find what lens placed at F^1 will correct this amount of astigmatism when near objects are being viewed. In other words what lens will give D_1 the value of D^1 .

We have $D_1 (a^2 - alp D) = p^2 D \dots \dots \dots (a)$

$$\therefore (a^2 - alp D) = \frac{p^2 D}{D_1}$$

But the corneal astigmatism is D^1 not D_1 , and $D^1 = \frac{D_0}{1 - l D_0}$

where D_0 represents the original cylinder (say $+4D$) which placed at F^1 corrected the astigmatism in distant vision.

$$\therefore a^2 - alp D = \frac{p^2 D}{D^1} = \frac{p^2 D}{D_0} - p^2 l D$$

$$\therefore D \left(\frac{p^2}{D_0} + pl \overline{a-p} \right) \text{ or } D \left(\frac{p^2}{D_0} + pl^2 \right) = a^2$$

$$\therefore D = \frac{a^2}{\frac{p^2}{D_0} + pl^2} \dots \dots \dots (3)$$

This is the general formula which is universally true, and gives correct results for convex or concave lenses according to the sign given to D_0 .

The table at the end gives the results to four decimal places when $p = 30\text{cm.}$ or 0.3m. , $l = 0.01375\text{m.}$, $a = 0.31375\text{m.}$

For the convenience of any one who cares to check my results I give the following values:

$$\begin{array}{ll} a^2 = 0.09843906 & pl^2 = 0.00005671875 \\ alp = 0.00129422 & p^2 = 0.09 \end{array}$$

The table is interesting in several ways. For instance, we have all noticed that aphakic patients who are lucky enough to see with $+11D$ the $6/9$ line, prefer $+15D$ for reading at $1/3\text{ m.}$ and not $+14D$; it is clearly simply an optical affair, the $+15D$ when used for reading has only the effectivity of $+14D$. Suppose that the best correction for a conical cornea is $-9D$ cyl. for distance, then $-10D$ cyl. will be the best cylindrical correction for reading.

It will naturally be supposed that sphero-cylinders will require a similar modification, but this is not so always. When testing a patient's refraction, we test it for distance, and then find by trial what additional spherical lens must be given to compensate for his presbyopia. Now if a full correction of $+3D$ is added for his presbyopia when reading at $1/3\text{ m.}$, it will be found that the table

gives quite erroneous results. In such a case + 3 D of the correction merely forms an image at a distance of the object (at 1/3 m. distance). Hence no modification in the cylinder will be necessary, for the cylinder already found by trial that corrects the astigmatism for distance will still correct the error for reading. But when no presbyopic addition is made, the table will be a useful guide to the alteration required. As a rough approximation one may say that if the cylinder that is found to correct the astigmatism at a distance be + 5 D :—

When the same spherical correction is used for near work the cylinder must be increased to + 5.45 D or say + 5.5 D.

When + 1 D is added to the spherical lens for near work, 2/3 of 0.45 should be added to the cylindrical lens theoretically, that is the cylindrical correction should be + 5.3 D.

When + 2 D is added to the spherical lens for near, 1/3 of + 0.45 D should be added to the cylindrical lens, or the cylindrical correction should be + 5.15 D.

When + 3 D is given for presbyopia, no addition to the cylindrical lens should be made, or + 5 D cyl. will be correct.

It will be well to work through an example in detail to show the correct procedure. Suppose + 4 D sph. $\ominus$ + 5 D cyl. is the accurate correction for distance; in one plane + 4 D is required and $D^1 = + 4.2328$, in the other plane + 9 D is required and $D^1 = + 10.2710$.

The actual corneal astigmatism is + 6.0382, the difference between the two values of D^1 . This corneal astigmatism is fixed and must be corrected for near work. Suppose first that no presbyopic addition is needed for reading. The effective value of + 4D at the cornea (D_c) when the object is 0.3m. from F^1 , is + 3.8601. This added to the corneal astigmatism is + 9.8983. We must now find what lens placed at F^1 will have this effective value at the cornea. Now on p. 10 from (a) it may be seen that

$$D = \frac{a^2}{p^2 + alp} \quad \text{As } \frac{p^2}{D_1} = \frac{0.09}{9.8983} = 0.00909247$$

$$\text{and } alp = \frac{0.00129422}{0.01038669}$$

$$\therefore D = \frac{0.09843906}{0.01038669} = 9.4774$$

Consequently the sphero-cylinder should have a power of + 4 D in one plane, and + 9.4774 in the other plane, *i.e.* practically + 4 D sph. $\ominus$ + 5.5 D cyl.

If a similar calculation be made for + 7 D sph. $\ominus$ + 5 D cyl., *i.e.* the accurate correction for distance, it will be found that if the

same spherical lens be used for work at 30 cm. the cylinder that will accurately correct the astigmatism for this distance will be + 5.499 D, or practically + 5.5 D again. If the case of - 7 D sph. $\ominus$ - 5 D cyl. be worked out in the same way, the cylinder required for near work will be found to be - 5.44 D instead of the previous - 5 D.

We will take this last case - 7D sph. $\ominus$ - 5D cyl., when a presbyopic addition of +3D is needed for work at 30cm.

For distance in one plane - 7D is required where $D^1 = -6.3854$
in the other plane - 12D where $D^1 = -10.3004$

The corneal astigmatism is therefore - 3.915, this added to the D_1 value of - 4D or - 3.4744 is - 7.3894, and it is found that

$$\begin{aligned} \text{as } \frac{p^2}{D_1} &= \frac{0.09}{-7.3894} = -0.01217961 \\ \text{and } \Delta p &= \frac{0.00129422}{-0.01088539} \\ \therefore D &= \frac{0.09843906}{-0.01088539} = -9.0432 \end{aligned}$$

The prescription - 4D sph. $\ominus$ - 5D cyl. can therefore be given with confidence for near work, as the 0.0432 is negligible. Indeed this small decimal fraction only arises because I have taken 30cm. as a more usual distance for reading than the orthodox 33 $\frac{1}{3}$ cm. If this distance ($\frac{1}{3}$ m.) had been taken, the +3D addition would have compensated for it exactly.

It will be seen that the value given in the table for +5D will indicate sufficiently closely the value of the cylinder required for near work when no presbyopic addition to the spherical component is made. The table was drawn up for a different purpose—to show the lenses which must be used for close work, if exactly the same ametropic correction is required as that for distance. Thus from the table + 7.6227 D, when used for work 30 cm. away, corrects the same amount of hypermetropia as + 7 D corrects at distance, and + 7.6227 D $\ominus$ + 5.404 D cyl. is equivalent to + 7 D sph. + 5 D cyl. at a distance. But although + 7 D sph. + 5.499 D cyl. when used for close work corrects + 5 D of astigmatism, it does not fully correct the + 7 D of hypermetropia.

The alteration in the spherical component given in the table may be neglected, but it will be seen that if mathematical accuracy is required a troublesome calculation is necessary. However for practical purposes any spherical addition can be found by trial, and it must be remembered that the length of the patient's arms has a good deal to do with the most comfortable distance for holding a book.

In the absence of a table it will be found that about 9 per cent. of the power of the lens required for distance must be added to

produce the same effectivity when used for close work, provided that no addition is made to the spherical component. Thus 0.45 must be added to a + 5D cyl. when used for work at 30cm. Hence if +1D is added for presbyopia the cylinder should be increased by about 6 per cent., if +2D is added, by about 3 per cent., and if +3D is added no increase of the cylinder is necessary.

There is one other point I may mention about reading glasses. When the eyes are converged and depressed, the superior obliques must play a very important part in the depression, and with this there may be a variable amount of intorsion depending upon the actual insertion of these muscles. If intorsion occurs the axes of the cylinders should be intorted also. Occasionally I have found that a slight intorsion of high cylinders results in better and more comfortable vision of near objects.

Distance.	30 cm. from F ¹ .	Distance.	30 cm. from F ₁ .
+ 1	+ 1.0031	- 1	- 1.0945
+ 2	+ 2.1848	- 2	- 2.1903
+ 3	+ 3.2751	- 3	- 3.2875
+ 4	+ 4.3641	- 4	- 4.3861
+ 5	+ 5.4517	- 5	- 5.4861
+ 6	+ 6.5379	- 6	- 6.5875
+ 7	+ 7.6227	- 7	- 7.6903
+ 8	+ 8.7062	- 8	- 8.7945
+ 9	+ 9.7884	- 9	- 9.9001
+ 10	+ 10.8692	- 10	- 11.0070
+ 11	+ 11.9486	- 11	- 12.1154
+ 12	+ 13.0267	- 12	- 13.2252
+ 13	+ 14.1034	- 13	- 14.3364
+ 14	+ 15.1788	- 14	- 15.4491
+ 15	+ 16.2529	- 15	- 16.5631
+ 16	+ 17.3256	- 16	- 17.6785

This table gives in the second and fourth columns the lenses that should be used for close work, if exactly the same correction is required as that for distance in the first and third columns.